# On Lipschitz Graphs on Two-step Carnot Groups

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We study classes of mappings of Carnot groups whose graphs are intrinsically Lipschitz, i. e., with respect to sub-Riemannian (quasi)metrics. Such distances correspond to the non-holonomic structure of the Carnot group, and they are not bi-Lipschitz equivalent to the distances defined by the Riemann tensor. Moreover, even in the model cases of graphs of smooth functions on a Carnot group, the resulting mappings are not intrinsically Lipschitz in general. So, it is differentiable neither in the classical nor in the sub-Riemannian sense.

In Classical Analysis and its generalizations, graph mappings play an essential role. For example, classes of minimal and maximal surfaces are locally representable as graphs (see, e. g., [1, 2] and references therein). In addition, at the beginning of the 21st century, a connection was found between problems in neurobiology on constructing visualization models and the properties of minimal surfaces in sub-Riemannian geometry (see [3, 4, 5], etc.). Thus, a natural question arises about the study of the existence and basic properties of graph mappings whose construction method is based on the group operation on Carnot groups, and which are intrinsically Lipschitz. This question also includes the description of classes of mappings that guarantee the Lipschitz property of their graphs.

The talk will include a criterion for mappings of two-step Carnot groups that ensure the Lipschitz property of their graphs. In particular, it turned out that the non-horizontal coordinates of the mapping must satisfy a certain differential equation depending on the structure constants of the Carnot group. In addition, we will give examples of structures on which non-trivial Lipschitz graphs do not exist. We will also describe examples of mappings that satisfy the properties derived in the criterion, whose graphs are Lipschitz. The results for Heisenberg and Carnot groups can be found in [6, 7].

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