G-convergence of operators with variable domain and applications to the asymptotic analysis of variational inequalities

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In this talk, we first recall an approach to the study of the convergence of solutions of variational inequalities with operators $A_s\colon W_s \to W_s^*$, where W_s is a real reflexive Banach space for any $s \in \mathbb{N}$. This approach developed in [1] involves the presence of certain relations of the spaces W_s with a limit Banach space W and the use of the notion of G-convergence of the considered operators, which is an analog of the corresponding notion for operators with the same domain (see, e.g., [3, 4]).

Then we show how the mentioned approach is realized for variational inequalities with elliptic operators $\mathcal{A}_s\colon W^{1,p}(\Omega_s) \to (W^{1,p}(\Omega_s))^*$ in divergence form, where $p > 1$ and Ω_s is a domain in $\mathbb{R}^n$ contained in a bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geqslant 2)$. As a result, we describe conditions for the convergence of solutions $u_s \in V_s$ of variational inequalities

$$\forall v \in V_s, \quad \langle \mathcal{A}_s u_s - f_s, u_s - v \rangle \leqslant 0,$$

where $f_s \in (W^{1,p}(\Omega_s))^*$ and V_s is a closed convex set in $W^{1,p}(\Omega_s)$. In particular, we consider constraint sets of the form

$$V_s = \{v \in W^{1,p}(\Omega_s) : \varphi_s \leqslant v \leqslant \psi_s \text{ a.e. in } \Omega_s\},$$

where $\varphi_s, \psi_s \in W^{1,p}(\Omega_s)$, $\varphi_s \leqslant \psi_s$ a.e. in Ω_s, and, more generally, of the form

$$V_s = \{v \in W^{1,p}(\Omega_s) : M_s(v) \leqslant 0 \text{ a.e. in } \Omega_s\},$$

where M_s is a mapping from $W^{1,p}(\Omega_s)$ to the set of all functions defined on Ω_s. For these cases, we show that under the G-convergence of the operators $\mathcal{A}_s$ to an invertible operator $\mathcal{A}\colon W^{1,p}(\Omega) \to (W^{1,p}(\Omega))^*$ and suitable assumptions on the domains Ω_s, obstacle functions φ_s and ψ_s, and the mappings M_s, solutions of the considered variational inequalities converge in L^p-norms to the solution of the corresponding limit variational inequality.

For our recent results on the convergence of solutions of variational inequalities with G-convergent operators and variable bilateral constraints in variable domains, see [2].

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